

Four-terminal SQUID:Magnetic Flux Switching in Bistable State and Noise

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Abstract

The effect of thermal fluctuations on the behaviour of a 4-terminal SQUID is investigated. The studied system consists of the Josephson 4-terminal junction with two terminals short-circuited by a superconducting ring and the other two form the transport circuit. The behaviour of a 4-terminal SQUID is controlled by external parameters, the applied magnetic flux and the transport current. They determine the voltage in the transport channel and the magnetic flux embraced in the ring. Within the numerical model the noise-rounded current-voltage characteristics and the time dependence of the magnetic flux have been calculated. In some region of the control parameters the 4-terminal SQUID is in the bistable state with two magnetic flux values. The switching of the flux in bistable state produced by the thermal noise or by the transport current is studied.

Keywords: Josephson junction; Multiterminal; SQUID; Thermal fluctuations

1 Introduction

Presently the properties of superconducting microstructures attract theoretical and experimental interest. Mostly it is due to the high level of technology which permits the fabrication of the ultrasmall Josephson weakly coupled systems with well defined geometry. Thus, the well known effects of weak superconductivity [1, 2] now can be studied in the situation approaching to the idealized theoretical models. From another side, new physics (e.g. unconventional proximity effect, reduced shot noise, Andreev's interferometry) appears in small mesoscopic structures [3]. The conventional Josephson effect in microconstrictions also can be considered as mesoscopic phenomena [4] produced by the macroscopic interference in the constriction of the initially coherent Cooper pairs wave functions of the two massive superconducting banks. The old idea [5, 6] of more complicated Josephson weak link structures, the so called Josephson multiterminals [7], was realized experimentally in ref. [8, 9]. In such structures the quantum interference takes place in the intersection of several microbridges connecting superconducting terminals. The specific multichannel interference effects were studied theoretically and experimentally in the novel superconducting device, the 4-terminal SQUID controlled by the transport current [10]. In this system we have the interplay between the coherent current states in the superconducting ring placed in the magnetic field and the Josephson effect in the transport circuit. The external transport current I and the applied magnetic flux Φ_e are the control parameters which govern the behavior of the 4-terminal SQUID. These two independent parameters determine the response of the system: the total magnetic flux Φ in the ring and the voltage V over the transport channel. It was shown [10] that in the steady state domain (when the transport current is less than the critical value $I_c(\Phi_e)$) the nonlinear parametric coupling leads to a bistable state of the SQUID $\Phi(\Phi_e, I)$ for all values of the self-inductance of the ring. Outside the steady state domain ($I > I_c$) the resulting flux Φ becomes an oscillating function of time whose amplitude, frequency and shape can be smoothly controlled by the external parameters [11, 12].

In the papers [10, 11, 12] the theory of a 4-terminal SQUID was developed without taking into account thermal fluctuations. The noise in the system obviously plays an essential role. In particular it leads to the magnetic flux switching in the bistable state, as was directly observed in experiment [13].

In the present paper we study the effects of thermal fluctuations on the

behavior of a 4-terminal SQUID. In Section 2, the basic equations used to describe the system are presented. The thermal noise is considered by adding the fluctuating voltages to the dynamical equations for the phases of the superconducting order parameter. These equations were solved numerically following the procedure described in [14]. In Section 3 the calculations of the noise-affected current-voltage characteristics for several values of applied magnetic flux are presented. The time dependence of the magnetic flux in the ring stimulated by noise is studied in Section 4.

2 Equations for the 4-terminal SQUID

The 4-terminal SQUID which is controlled by the transport current is shown in Fig.1. It consists of the Josephson 4-terminal junction with two terminals (3 and 4) short-circuited by a superconducting ring and the other two (1 and 2) form the transport circuit.

The set of equations describing the behavior of a 4-terminal SQUID in an applied magnetic flux and a transport current was obtained in ref.[10]. The Josephson supercurrent in the j th filament (connecting the centre o with the j th bank) expressed in terms of the phases of the superconducting order parameter (related to the phase in the center o) has the form

$$I_j^s = \frac{\pi \Delta_0^2(T)}{4ekT_c} \frac{1}{\sum_{k=1}^4 1/R_k} \sum_{k=1}^4 \frac{1}{R_j R_k} \sin(\varphi_j - \varphi_k) \quad (1)$$

Here Δ_0 is the gap in the bulk banks, R_j is the normal resistance of the j th filament. Let V_j be the voltage in j th terminal, related to the voltage in the centre ($\sum_j V_j/R_j = 0$). In the frame of the heavily damped resistively shunted model[2] we add to supercurrents (1) the normal currents $I_j^n = V_j/R_j$ with $V_j = \frac{\hbar}{2e} \dot{\varphi}_j$. Thus for the total current flowing in j th branch we have the expression in terms of the phases φ_j

$$I_j = \frac{\pi \Delta_0^2(T)}{4ekT_c} \frac{1}{\sum_{k=1}^4 1/R_k} \sum_{k=1}^4 \frac{1}{R_j R_k} \sin(\varphi_j - \varphi_k) + \frac{\hbar}{2eR_j} \frac{d\varphi_j}{dt} + \frac{\delta V_j(t)}{R_j}, \quad (2)$$

$j = 1, 2, 3, 4.$

The positive sign of I_j (2) corresponds to the direction of the current from the centre to j th bank. Note that in the resistively shunted model the superconducting and normal currents are conserved separately and equations (2) satisfy to conservation of the total current, $\sum_{j=1}^4 I_j = 0$.

The third term in the r.h.s. of equation (2) represents the thermal Johnson noise of the normal resistance R_j . We assume that the time dependent voltage noise sources are uncorrelated, each having a white voltage spectral density $S_V = 4kTR_j$.

The values of the currents I_j are determined by the circuit implication of the Josephson multiterminal. In the 4-terminal SQUID configuration considered here (see Fig.1) they are connected to the external controlling parameters, the given transport current I and the applied magnetic flux Φ_e , in such a way that

$$I_1 = -I, I_2 = I, I_3 = -J, I_4 = J, J = (\Phi_e - \Phi)/L, \quad (3)$$

where J is the circulating current in the ring with self-inductance L on which a magnetic flux Φ_e is applied; The total embraced flux Φ is related to the phase difference between the terminals 4 and 3

$$\Phi = \frac{\hbar}{2e}(\varphi_4 - \varphi_3). \quad (4)$$

Four equations (2) with the relations (3) and (4) constitute the coupled system of differential equations for the phases φ_j and describe the dynamical behavior of the 4-terminal SQUID in the presence of thermal noise. They have the integral of motion $\sum_j \dot{\varphi}_j / R_j = C$, where the value of the constant without loss of generality can be put to zero. Thus we have three independent phases and it is convenient to introduce new variables

$$\varphi_2 - \varphi_1 = \theta, \varphi_4 - \varphi_3 = \phi, \frac{1}{2}(\varphi_1 + \varphi_2) - \frac{1}{2}(\varphi_3 + \varphi_4) = \chi. \quad (5)$$

In the following for simplicity we will consider the symmetric case $R_1 = R_2 = R_3 = R_4 = R/2$. We use the following dimensionless units: current in units of $I_0 = \pi\Delta_0^2/(4ekT_cR)$, voltage in units of I_0R and time in units of $\hbar/(2eRI_0)$. The dynamical equations for the variables θ, ϕ, χ take the form

$$\begin{aligned}
\frac{d\theta}{dt} &= I - \frac{1}{2} \sin \theta - \sin \frac{\theta}{2} \cos \frac{\phi}{2} \cos \chi + \delta v_\theta(t), \\
\frac{d\phi}{dt} &= \frac{\phi_e - \phi}{\mathcal{L}} - \frac{1}{2} \sin \phi - \sin \frac{\phi}{2} \cos \frac{\theta}{2} \cos \chi + \delta v_\phi(t), \\
\frac{d\chi}{dt} &= -\sin \chi \cos \frac{\phi}{2} \cos \frac{\theta}{2} + \delta v_\chi(t),
\end{aligned} \tag{6}$$

where $\phi_e = \frac{2e}{\hbar} \Phi_e$ and $\mathcal{L} = \frac{2e}{\hbar} I_0 L$ is the dimensionless self-inductance. The "random forces" $\delta v_\theta, \delta v_\phi$, and δv_χ are related to the initial dimensionless fluctuating voltages in eq.(2) :

$$\delta v_\theta = \delta v_1 - \delta v_2, \delta v_\phi = \delta v_3 - \delta v_4, \delta v_\chi = -\frac{1}{2}(\delta v_1 + \delta v_2) + \frac{1}{2}(\delta v_3 + \delta v_4). \tag{7}$$

The corresponding spectral densities in dimensionless units are

$$S_{v_\theta} = S_{v_\phi} = 4\Gamma, S_{v_\chi} = 2\Gamma, \Gamma = \frac{2ekT}{\hbar I_0}. \tag{8}$$

The system of the coupled equations (6) determines the time dependencies of the phases $\theta(t), \phi(t)$ and $\chi(t)$ in the presence of thermal noise at given values of the control parameters - the transport current I and the external magnetic flux ϕ_e , which are assumed to be independent of time. The time derivatives of the phase differences determine the voltages between different terminals:

$$V_{21} = \dot{\theta}, V_{43} = \dot{\phi}, \frac{1}{2}(V_{13} + V_{24}) = \dot{\chi}. \tag{9}$$

3 Current-voltage characteristics in the presence of noise.

We solve the equations (6) numerically by integrating the phases stepwise in time with a step Δt . The Johnson noise voltages $\delta v(t)$ are modelled by three uncorrelated trains of voltage pulses of constant duration Δt and amplitudes δv . The amplitudes δv are Gaussian-distributed pseudo-random numbers of zero mean value with $\langle \delta v_\theta^2 \rangle = \langle \delta v_\phi^2 \rangle = 2\Gamma/\Delta t$, $\langle \delta v_\chi^2 \rangle = \Gamma/\Delta t$ in accordance with the spectral densities (8). The value of Γ (8) characterizes the noise level in our system.

In this section we use the above formulated numerical model to calculate the noise-affected current-voltage characteristics of the 4-terminal SQUID. The observable quantity V , the voltage in the transport channel averaged over time, was obtained from the solution of eq.(6) for the phase $\theta(t)$: $V \equiv \langle \frac{d\theta}{dt} \rangle_t$.

To check the numerical techniques we first calculated numerically the noise-rounded IV characteristics for the single resistively shunted Josephson junction. In this well known case the phase θ satisfy $\dot{\theta} = I - \sin \theta + \delta v_\theta(t)$ and from the corresponding Fokker-Planck equation the average voltage across the junction was obtained by Ambegaokar and Halperin [15] analytically. In Fig.2 we compare our numerical results with the Fokker-Planck calculation, they are in good agreement.

Noise-rounded IV characteristics calculated from the full set of equations (6) for two values of applied magnetic flux $\phi_e = 0, \pi$ are displayed in Fig.3 a,b. We choose $L=1.7$, $\Gamma = 0.2$, close to the values estimated in experiments [13]. Figure 4 illustrates the behavior of current-voltage characteristics in an applied magnetic field. For given value of the transport current I' and changing of the magnetic flux Φ_e the average voltage periodically oscillates with amplitude δV . For the detailed analysis of this behaviour in the fluctuation free case see ref. [12].

4 Magnetic flux switching in the bistable state due to the noise and transport current.

We now use the numerical model to study the behaviour of the embraced magnetic flux in the ring.

Let us start with the fluctuation free case. For the time independent control parameters I and ϕ_e , we can introduce a potential:

$$U(\theta, \phi, \chi | I, \phi_e) = \frac{(\phi - \phi_e)^2}{2\mathcal{L}} - I\theta - \cos^2 \frac{\theta}{2} - \cos^2 \frac{\phi}{2} - 2 \cos \frac{\theta}{2} \cos \frac{\phi}{2} \cos \chi \quad (10)$$

such that the dynamical eqs.(6) take the form

$$\dot{\theta} = -\frac{\partial U}{\partial \theta}, \dot{\phi} = -\frac{\partial U}{\partial \phi}, \dot{\chi} = -\frac{1}{2} \frac{\partial U}{\partial \chi} \quad (11)$$

The stable steady states of the system correspond to the minima of potential U with respect to variables θ, ϕ, χ at given values of I and ϕ_e . The minimization of U with respect to χ gives that phase χ takes the value 0 or π , depending on the equilibrium values of θ and ϕ : $\cos \chi = \text{sign}(\cos \frac{\theta}{2} \cos \frac{\phi}{2})$. The stable configurations of the noise-free 4-terminal SQUID were studied in ref. [10]. It was shown that at the steady-state domain in the plane of (I, ϕ_e) the region exists, inside which the potential (10) has two minima (see Fig.3 in ref.[12]). The structure of this bistable state, the height and the width of the potential barrier in the 3-dimensional phase space (θ, χ, ϕ) can be regulated by control parameters I and ϕ_e . For example, at $\phi_e = \pi$ we have two equilibrium values of induced magnetic flux, which are equally located around π : $\phi_{1,2}(I) = \pi \pm \Delta\phi$. The current circulating in the ring in this case equals $\pm j$. The values of the potential U at these two states are equal, $U_1(I, \phi_e = \pi) = U_2(I, \phi_e = \pi)$, and in which state the SQUID is located depends on the history of the system. It is emphasised that in contrast to the case of the usual SQUID, the described bistable state of the 4-terminal SQUID exists for each value of inductance \mathcal{L} even for $\mathcal{L} < 1$.

The presence of noise, *i.e.* the three random forces $\delta v(t)$ in the dynamical eqs.(6), produces small fluctuations of phases ϕ, θ, χ near the equilibrium values, as well as transitions between the two states. Each such transition corresponds to the switching of the magnetic flux ϕ between the two values ϕ_1 and ϕ_2 . The switching of magnetic flux must be accompanied by a switching of χ with a value $\pm\pi$ or by a 2π slippage of θ . Note, that the phases θ and χ are independent parameters, having different physical meaning. In accordance with the eqs.(9) the dynamics of θ determines the voltage in the transport channel, while the time dependence of χ means the appearance of the voltage between the ring and the transport circuit. In other words, the last one means the electrical charging of the ring. The random event of one from the two possible processes, switching of θ or χ , is determined by the magnitudes of the corresponding potential barriers ΔU_θ and ΔU_χ . They depend on the value of the transport current I . By analyzing the shape of the potential $U(\theta, \phi, \chi | I, \phi_e)$ surfaces can qualitatively be concluded (see ref.[13]) that for small values of I the transitions with switching of χ are more probable than slippage of θ , but with the increasing of the current I the

situation will change to the opposite.

To study the processes of switching in the bistable state, produced by the thermal noise, we numerically solve the eqs.(6) and obtain the time dependencies $\phi(t), \theta(t), \chi(t)$. Figs.5 and 6 show some typical traces of the phases ϕ, χ and θ as a function of time t at an external flux value $\phi_e = \pi$ for particular values of the noise parameter $\Gamma = 0.2$ and of the selfinductance $\mathcal{L} = 1.7$.

Fig.5 shows the behaviour for the transport current $I = 0$. In this case we observe the noisy behaviour of phases and the random switching of ϕ between two flux states. When ϕ switches, the phase χ jumps at the same moment in time, with a value of $\pm\pi$. In the same time, although the behaviour of $\theta(t)$ is noisy, no switching is observed in θ .

In Fig.6 the transport current equals $I = 0.25$. It displays the case, in which the switching of ϕ between both flux states may be accompanied by a jump of χ with a value of $\pm\pi$ or a 2π phase slippage of θ . In addition, in the numerical simulations the case is observed, that χ and θ jump at the same time while ϕ remains constant.

The behaviour observed in Figs.5 and 6 corresponds to the above mentioned dependence on the transport current of the potential barriers ΔU_θ and ΔU_χ . The switching of the magnetic flux can be induced not only by the noise but also by the transport current I , if it exceeds the critical value $I_c(\phi_e)$. Fig.7 shows the time dependencies of the phases ϕ, χ, θ in the noise-free case ($\Gamma = 0$) for the value of the current $I = 1.05$. This value only very slightly exceeds the critical current (see Fig.3) and we see that the flux ϕ adiabatically follows $\theta(t)$, except the moments of time when the slippage of θ takes place and the corresponding switch in the flux occurs. In the same time, in this regime, the phase χ remains constant.

5 Summary

The four terminal SQUID presents a system with three degrees of freedom, the phase differences ϕ, θ, χ . In the steady state domain the observable quantity is the magnetic flux embraced in the ring, $\Phi = \frac{\hbar}{2e}\phi$. In some region of the independent control parameters, the transport current I and the applied magnetic flux ϕ_e , the system is in the bistable state with two magnetic flux values. The operation of the four terminal SQUID, *i.e* the choosing of one

from two possible states, can be achieved by variation of I and ϕ_e . At fixed values of the control parameters, the magnetic flux switching is produced by the thermal noise or by the transport current, which exceeds the critical value.

We have computed the transitions between two flux states stimulated by the thermal fluctuations. The switches of the flux are accompanied by the jumps in θ or χ and the corresponding voltage impulses. The existence of a bistable state in a four terminal SQUID was experimentally confirmed in ref.[13]. by the direct observations of the magnetic flux behaviour, which is similar to the results obtained numerically in Section 4.

References

- [1] B.Josephson, Adv.Phys. **14** (1965) 419.
- [2] A.Barone, G.Paterno, Physics and Applications of the Josephson Effect, Wiley, New York, 1982.
- [3] Y.Imry, Introduction to Mesoscopic Physics, Oxford,New York,1997,
- [4] P.F.Bagwell, Physica B 203 (1994) 475.
- [5] K.K.Likharev, Radio Techn.Electr. 20 (1975) 660; Rev.Mod.Phys. 51 (1979) 101,146 and 147.
- [6] I.O.Kulik, A.N.Omelyanchouk, E.A.Kel'man, Sov.J.Low Temp.Phys. 5 (1979) 55
- [7] E.D.Vol, A.N.Omelyanchouk, Low Temp.Phys. 20 (1994) 87.
- [8] B.J.Vleeming, A.V.Zakarian, A.N.Omelyanchouk and R.de Bruyn Ouboter, Physica B 226 (1996) 253; Czechoslovak Journal of Physics **46**,S5 2823 (1996).
- [9] A.W.Fortuin, PhD thesis Technical University Delft (1997).
- [10] R. de Bruyn Ouboter, A.N.Omelyanchouk, E.D.Vol, Physica B 205 (1995) 153.
- [11] E.D.Vol, A.N.Omelyanchouk, Low Temp.Phys. 22 (1996) 742.
- [12] R. de Bruyn Ouboter, A.N.Omelyanchouk, E.D.Vol, Physica B 239 (1997) 203.
- [13] B.J.Vleeming, F.J.C.van Bemmelen, M.R.Berends, A.N.Omelyanchouk and R. de Bruyn Ouboter, to be published in Physica B.
B.J.Vleeming, PhD thesis, Leiden University (1998).
- [14] C.D.Tesche, J.Clarke, J. Low Temp. Phys. 29 (1977) 301.
- [15] V.Ambegaokar, B.I.Halperin, Phys.Rev.Lett. 22 (1969) 1364.

FIGURE CAPTIONS

Figure 1. The 4-terminal SQUID. The area of the region closed by the dashed lines (the Josephson 4-terminal junction) is of the order of the coherence length squared.

Figure 2. A comparison of the numerically calculated noise-rounded IV characteristics (points) for a single resistively shunted junction with the analytical results from ref.[15] (solid lines). The dashed line is the noise-free characteristic.

Figure 3. Current- voltage characteristics of the current-biased transport circuit of a 4-terminal SQUID in the presence of noise for $\Gamma = 0.2, \mathcal{L} = 1.7$ and for two external flux values $\phi^e = 0$ (a) and $\phi^e = \pi$ (b). The solid lines are the noise-free characteristics.

Figure 4. The changing of the noise-rounded current-voltage characteristic of a 4-terminal SQUID in an applied magnetic field. δV shows the amplitude of the voltage oscillations as a function of the external flux Φ^e for an applied transport current I' .

Figure 5. The three phase differences ϕ , χ and θ versus time t for $\phi^e = \pi, \mathcal{L} = 1.7$ in the presence of noise with $\Gamma = 0.2$. The transport current $I = 0$. Transitions between the two flux states occur (jumps in ϕ) which are accompanied by jumps in χ by $\pm\pi$. At zero transport current no switching in θ occurs.

Figure 6. This figure can be compared with Fig.5 but now the transport current is equal to $I = 0.25$. Again $\phi^e = \pi$. This is an intermediate situation where the switching of ϕ is accompanied by jumps in χ by $\pm\pi$ or in θ by 2π . At higher currents the χ -fluctuation mechanism will eventually be taken over by the θ -fluctuation mechanism.

Figure 7. The time dependencies of the phase differences ϕ , χ and θ in the noise-free case ($\Gamma = 0$) for the value of the current $I = 1.05$, which slightly exceeds the critical current and for $\phi^e = \pi$. At the moments of time when the 2π phase slippage of θ takes place the switch in the flux ϕ occurs. The phase χ in this regime remains constant.